# On the Materiality and Structural Behaviour of highly-elastic Gridshell Structures

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### Abstract

Gridshell structures made of highly elastic materials provide significant advantages thanks to their cost-effective and rapid erection process, whereby the initially in-plane grid members are progressively bent elastically until the desired structural geometry is achieved. Despite the strong growing interest that architects and engineers have in such structures, the complexity of generating grid configurations that are developable into free-form surfaces and the limitation of suitable materials restrict the execution of elastically bent gridshells.

Over the past ten years, several research studies have focused on methodologies to generate developable grid configurations and to calculate their resulting geometry after the erection process. However, the same curved shell surface can be reproduced by various developable grid configurations which, in combination with their material properties, exhibit different structural behaviours not only during the shaping process but also on the gridshell load-bearing capacity.

In this paper, the structural consequences of the choice of the grid configuration for an anticlastic surface have been analysed by means of FEM-Modelling combined with an geometrical optimisation of the initial bending stresses. In addition, the potential of using natural fibre-reinforced composites as a lightweight and environmentally friendly alternative has been investigated.

### 1. Introduction

Research works focused on elastically shaped gridshell structures have intensified during the last decade. Since the first gridshell structure, the Multihalle in Mannheim, Germany (1975), was successfully built, diverse methodologies have been developed which calculate the curved gridshell geometry resulting from the shaping process. The complexity of the calculation relies on the determination of the bending moments induced by the erection process and conditioned by the evolution, deformation and distortion, of the initial grid configuration.

Due to the resultant bending in the profiles, the highest material utilisation in highly-elastic gridshells usually occurs during the erection process. The induced bending moments (1) are directly proportional to the material's Young's modulus and cross-section properties (second moment of area) and inversely proportional to the gridshell curvature (radius of curvature of the grid profiles).

$$M_B = EI / r \qquad (1)$$

In the following figures (Fig. 1), the von Mises stresses of an elastically bent anticlastic gridshell are shown. The gridshell on the left, which is not yet subjected to external loading, already exhibits a material utilisation of about 60%. When an uniformly distributed snow load of 0.9 kN/m$^2$ is applied to the gridshell, the utilisation percentage increases to approximately 90%.

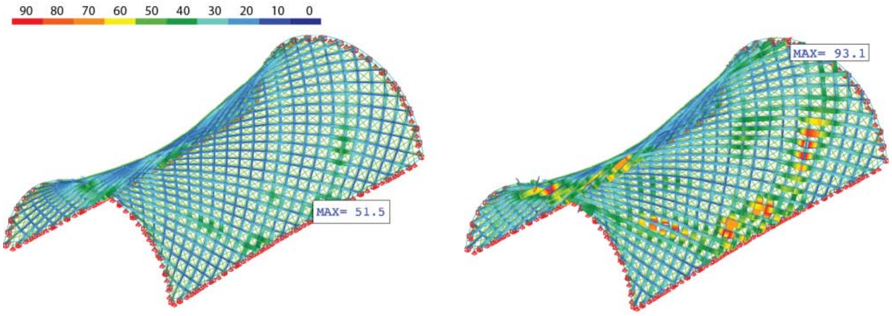


*Fig.1 Von Mises bending stresses of an anticlastic gridshell*
*Without external loads (left), under uniformly distributed snow (right)*

## 2. Gridshells - Materials of Choice

When choosing materials for highly elastic gridshells, two important aspects should be considered. The first is the material's bending elasticity. The Young's modulus of the chosen material should be sufficiently low in order to minimise bending moments which result from shaping the shell and, consequently, the erection forces during the construction phase as well as the reaction forces at the supports. Conversely, the Young's modulus should be high enough in order to provide the sufficient global buckling stiffness. With high ultimate strains, the profiles are able to be further bended before the maximum stresses are attained.

The second relevant aspect is the material strength capacity. Materials with higher ultimate stresses can resist major bending moments and, hence, are able to adopt a lower radius of curvature and, consequently, are more appropriate for gridshells with strong curvature. Therefore, the choice of the material should not be independent of the gridshell in question, but it should consider its geometric as well as structural requisites.

Moreover, the material selection also affects the definition of the cross-section. While for example timber beams are restricted to solid closed sections, the manufacture of pultruded composite materials offers a great variety of hollow-sections. Here, the shape and wall thickness of the cross-sections can be optimised such that bending stresses during the construction process can be minimised and sufficient global stability of the shaped gridshell can be achieved.

## 2.1 Existing Materials

The Multihalle in Mannheim, Germany (1975) [1] and the Weald and Downland Museum [2] in Sussex, Great Britain (2002) were both pioneering examples of gridshell structures and both made use of timber, hemlock and oak. With a Young's modulus of about 10 GPa, a bending strength of about 30 MPa and an ultimate strain of about 2%, these timber materials required two profile layers in each grid direction. During the construction process, each layer was bent independently from each other, so that the bending stiffness of only one layer had to be overcome. Once the gridshell was shaped, shear blocks were added between the two layers, in order to transfer shear forces between them. With it, the bending stiffness of both layers together was activated providing the structure with higher rigidity and global stability.

Emerging composite materials such as glass fibre-reinforced plastics (GRP), with higher ultimate strength properties (300 - 400 MPa) and modulus of elasticity (20 - 40 GPa) than timber, are able to afford more rigidity to the structure, so that, instead of two layers of profiles, only one in each grid direction is needed, which facilitates and accelerates construction. The first studies on GRP gridshells were carried out by the Institut Navier, France (2005) [3]. During the shaping process, the bending stresses tend to be higher than by timber, as GRP has a higher rigidity, but this is compensated by a higher ultimate strength, which allows a further bending of the profiles before the maximum stresses are attained. Another advantage of composite materials is that arbitrary profile lengths can be manufactured thanks to pultrusion technology and thereby weaknesses otherwise caused by nodal connections can be avoided.

Despite the good mechanical properties of GRP, the high environmental impact of the glass fibre production and the difficulties in recycling and reuse have always been critical aspects. Retaining the advantages of composites, natural-fibre reinforced plastics (NFRPs) promise to be a more environmentally friendly and lightweight alternative to GRPs.

The following table presents the mechanical bending properties of structural timber (D30, according to Eurocode 5), GRP and NFRP [4]. NFRP, with a rigidity value in-between that of timber and GRP, has a bending strength more than three times higher than timber (*Fig. 2 - left*). Compared to glass-fibre reinforced plastics (GRP), NFRP properties become more interesting when regarding the values with respect to their weight-ratio (*Fig. 2 - right*). By choosing appropriate fibre volume fractions, competitive properties can be achieved.

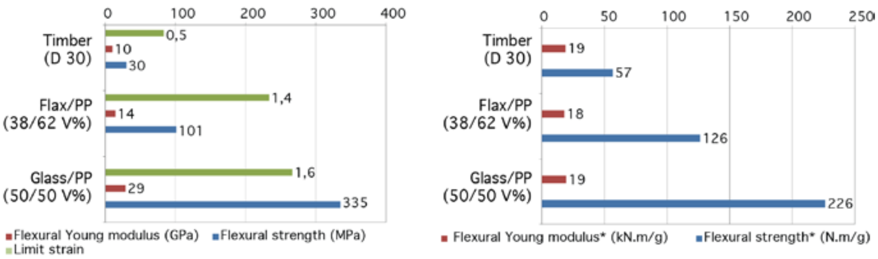


*Fig. 2 Comparison of timber, NFRP and GRP mechanical properties*

### 2.2 NFRP as Building Material

At present the demand for sustainable construction is becoming increasingly strong. One strategy for more resourceful and environmentally-friendly construction is the selection of materials and systems in terms of their life cycle assessment (LCA) and environmental impact. A further strategy is to apply the principle of building with lightweight materials and structures. In general, materials made from renewable raw materials offer strong performance in terms of sustainability and environmental impact.

NFRPs, based on rapidly renewable fibres and biobased and/or biodegradable matrices, can be used for the economically competitive and industrially scaled manufacture of unidirectional fibre-reinforced profiles by means of the pultrusion technique. NFRP profiles with a unidirectional orientation of fibres offer ideal mechanical properties for a variety of structural applications.

As mentioned above, global stability is one of the most relevant structural aspects of lightweight gridshells. Generally, it can be said that buckling loads are directly proportional to the bending stiffness of the grid profiles. Let us consider four different cross-sections (*Fig. 3)* each with the material properties of timber, NFRP and GRP respectively and each having the same bending stiffness (EI). The first timber cross-section corresponds to the double rectangular layer used in the Weald and Downland Museum, while the second cross-section to a classic rectangular timber profile, with the same proportions than the rectangular profiles used on the first section. Tubular cross-sections have been chosen for the composites materials, as fibre-reinforced hollow sections are feasible with pultrusion and because they offer further advantages in terms of optimisation of profile weight and cross-sectional properties.

One can see that, with less material quantity, single-layer pultruded sections are able to achieve stiffness properties equivalent to the double-layer timber system, thanks to the efficiency of hollow sections combined with the composite’s higher modulus of elasticity. The NFRP tube is the lightest alternative, with a section weight corresponding to 87% and 80% that of the timber and GRP profiles respectively. As expected, the timber single-layer section is the most inefficient option, being near three times heavier than the NFRP profile.

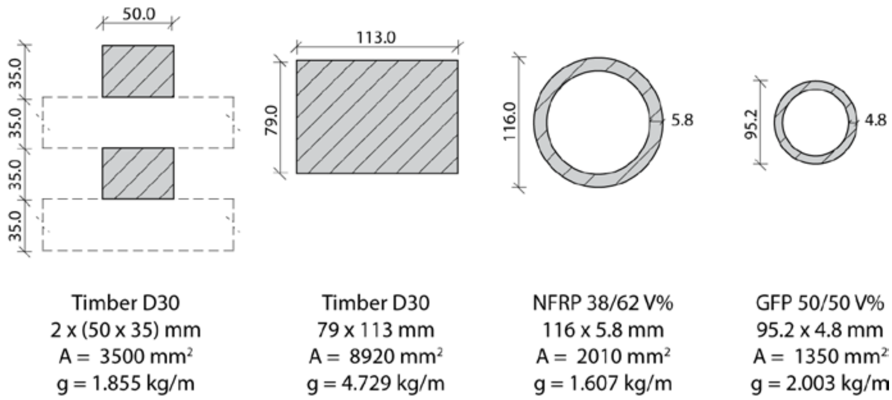


*Fig. 3 Timber, NFRP and GRP cross-sections with equivalent bending stiffness*

Nevertheless, the considered cross-sections also differ in the ultimate strength capacities and, consequently, in the utilisation rate induced by the gridshell shaping as well as in the ultimate bearing moment under external loads, both depending on the surface curvature of the resulting gridshell. Generally, the higher the material's mechanical strength, the later the ultimate stresses are reached and, consequently, the further the profiles are able to be bent, making possible new geometries and forms.

Depending on the gridshell geometry and the service loads that the structure will be subjected to, the considered materials become more or less appropriate. The optimal material and cross-section for a specific gridshell structure can be obtained when both aspects, global stability and the material's ultimate strength, are considered.

## 3. Gridshells - Shaping of Developable Grids

Gridshell geometry results from a shaping process, where an initially in-plane two-directional grid configuration is progressively elastically bent until the desired gridshell curvature is obtained. The profiles in the first direction are connected to the profiles in the second direction by pin joints which allow the distortion of the grid that is required to reproduce the target surface geometry. After the shaping process and before removing the shaping external forces, the edges of the gridshell will be fixed by rigid border beams and diagonal bracing will be added in order to maintain the desired geometry, otherwise the grid would further distort by searching an internal equilibrium, and to obtain in-plane shear bearing capacity.

Formfinding methods are required to generate grid configurations developable on specific surface geometries and to determine the internal forces and moments after shaping. Various studies have developed methodologies which can be basically classified according to the process with which the grid configurations (profile orientation and lengths) are defined. In some methods the grid is geometrically determined, while in other methods the grid results from an equilibrium of forces after applying external shaping forces to the modelled structure.

Nevertheless, the same target surface geometry can be reproduced by numerous grid configurations, differing in the profile lengths and the angle between both grid directions, once the gridshell is already shaped. The grid configurations have an important influence not only on the initial bending stresses induced during the erection process, but also on the structural behaviour of the gridshell under external loading. In this chapter, three different configurations of an anticlastic gridshell have been analysed in order to identify the structural consequences of each grid choice.

### 3.1 Existing Formfinding Methods

Existing formfinding methods can be classified into two groups depending on how the developable grid is defined: grids geometrically defined and grids resulting from equilibrium of forces.

Methods with a geometric definition of the grids include:

- *The 'compass' method*

In 1974 Frei Otto’s Institute for Lightweight Surface Structures (Institut für leichten Flächentragwerke) [5] proposed one geometric method consisting of tracing a grid with equilateral meshes on a target surface, at that time with only the help of a compass. One should start by defining on the surface two arbitrary main curves, the main profile directions, having a common intersection point. Dividing these main curves into equal segments, corresponding to the desired mesh size, one can consider the intersection point and the endings of the first adjacent segments of both main curves as three corners of the first equilateral mesh. The fourth corner can be then determined by tracing circles on the surface with a radius equal to the mesh size and centres at the two segment endings. The intersection of both circles gives the fourth mesh corner. The following meshes can be traced by using the same principle.

In 2007 M.H. Toussaint of the Delft University of Technology [6] developed a design tool based on the same principle using the three dimensional modelling CAD software Rhinoceros$^{TM}$. The tool consists of a script capable of automatically generating all the grid meshes on any target surface, once the main curves are given. Instead of circles, spheres are used to find the intersection points of the grid profiles. Knowing the coordinates of these points, the tool can also check the profiles’ curvatures.

Although the latter method can be used to generate grids developable into specific surfaces, an additional calculation, taking into account the internal forces induced by the bending process, is required to determine the resulting gridshell geometry.

- *Dynamic Relaxation with initial plane geometry*

This method was used for the design of the Weald and Downland Museum in Sussex, Great Britain (2002) [2] and the GFP gridshell in Navier Institut, France (2005) [3]. With the dynamic relaxation method the resulting gridshell geometry in equilibrium after shaping can be calculated. This method consists on defining, the movement of the profile intersection points, modelled as nodes with a certain fictitious mass, of a pre-defined plane grid configuration, applying Newton’s 2$^{nd}$ Law and resolving the equilibrium of forces in each node. In this equilibrium of forces, the internal forces correspond to the axial forces and bending moments generated during the construction process and the external forces to the shaping ones.

Although this method provides the effective curved gridshell geometry, the starting plane nodes configuration, and respectively the initial flat geometry, are geometrically predefined. For the Weald and Downloand Museum a squared flat grid of 49 m long and 24.2 m wide was used, while for the GFP gridshell an elliptic plane geometry was chosen.  A specific target surface is difficult to be achieved with this method.

Methods with grids resulting from the application of a system of forces are:

- '*Hanging chain' method*

The physical *hanging chain* method was employed for the design of the first built gridshell structure, the Multihalle in Mannheim, Germany, in 1975 [1]. Neglecting the bending stiffness of the grid profiles, the *Institut für leichte Tragwerke* with Frei Otto, studied the possibility of reproducing the resulting curved gridshell geometry by means of a suspended net model with equilateral meshes. The connections between the grid profiles were modelled as hinges using rings. In order to achieve the desired surface curvatures, the number of chain members was manually modified until the final gridshell geometry was obtained. Finally, the resulting position of the nodes were redetermined by photogrammetry and, with it, the static equilibrium of the final structure was calculated using the force density method and by considering the real weight of the gridshell.

One can consider that the grid configuration results as an iterative visual process influenced by the resulting net geometry due to suspension forces. Although physically generated, equilibrium of forces help to define the final grid. With this method the effective curved gridshell geometry cannot be generated precisely, since the internal bending moments and forces, relevant for the structural equilibrium of the gridshell, are neglected.

- *Dynamic Relaxation with shape approximation*

In the design methodology developed by M. Kuijvenhoven from the Delft University of Technology (2009) the grid configuration results from an approximation process, which generates a grid as close as possible to a target surface without exceeding the material's permissible stresses [7]. The curvature of the profiles is controlled by a system of transverse springs which are linked to and migrate towards the initial target grid geometry. The spring coefficients, and consequently the shaping forces, are modified iteratively so that at any point in the grid the permissible stresses are not exceeded. Once the approximate grid geometry is defined, the final gridshell geometry, modelled by a second system of interpunctual and rotational springs, is calculated by removing the action of the shaping forces. Dynamic relaxation is used here to find geometries in equilibrium, firstly, during the approximation phase and, secondly, after removing the shaping forces.

The limitations of the tool are that the grid configuration can only be modified approaching the grid vertically towards the target surface, that only one grid orientation can be considered for the shape approximation and that torsion and shear are not taken into account when calculating the resulting gridshell geometries.

- *Dynamic Relaxation with application of a vertical system of forces*

In 2009 the Navier Institute, France, proposed a second methodology where firstly the initially flat grid is set up over the target surface andsecondly a system of vertical forces, resulting from a convergence analysis, is applied to the rearranged grid so that this one is able to acquire the desired geometry [8]. The equilibrium shape of the gridshell is calculated using the dynamic relaxation method, permitting friction between the grid and the target surface. Unlike the shape approximation method, no optimisation of the post-shaping bending stresses is performed.

### 3.2 Finite Element Modelling with optimisation of the initial bending stresses

In the methods described above, the resultant grid geometry after formfinding generally depends on the choice of the initial grid orientation. Indeed, the same target surface can be reproduced by various grid configurations, which basically differ from one another in the resulting angle between the profiles and their lengths. In order to analyse the structural consequences of the choice of grid type and to determine the most appropriate grid configuration, a design methodology has been developed which takes into account the curvature of the bent grid profiles of a target surface and the load-bearing capacity of the resulting gridshell. This method is explained below:

Firstly, the grid configuration will be generated by means of an algorithmic calculation which tends towards minimized curvatures of the bent profiles and consequently minimized bending stresses during the shaping process. For example, a 30m long and from 14 to 15m wide anticlastic surface is considered. Three different grid configurations were generated, with acute, rectangular and obtuse angles between the grid profiles in the transverse direction, respectively. The corresponding plane geometries can be determined using three dimensional CAD.

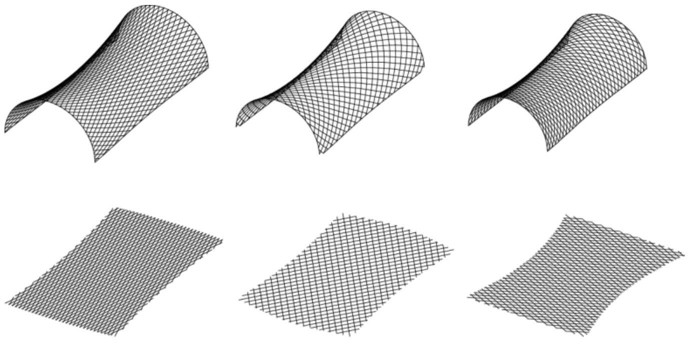


*Fig. 4 Grid configurations acquiring the same anticlastic surface*
*Plane (above) and curved geometries (below)*

Secondly, the shaping process of the optimized grid configurations is reproduced by means of finite element analysis in order to obtain resultant gridshell geometry and to evaluate the structural behaviour under external loading. One advantage of the finite elements methods is that bending, torsion as well as shear are modelled.

*Minimisation of the Initial Bending Stresses*

The algorithm minimizing the curvature of the profiles consists of defining energies that penalise deviation from an optimum. In our case we need to minimize three variables at once. Firstly, the curved grid configuration must remain close to the target surface, so that the positions of the profile intersection points are constrained to stay in the vicinity of the reference surface. Secondly, in order to generate a developable grid, the mesh size should be constant. And thirdly, the angle between two consecutive profile segments should be, theoretically, almost 180° in order to avoid bending stresses. This results in a linear combination of three *energies* and their corresponding coefficients.

The energies in question are typically non-convex. This means that there are many local minimizers which an algorithm might find. The key to finding a good geometry with the desired properties is now the initialization of the minimization. Once an initial guess is found, a gradient descent or the Newton method can be applied to find local minimizers of the function. In our case the initialization is found by starting with a remesh stemming from a conformal parametrization, thus having minimal geodesic curvature. From here we minimize the described function to achieve a uniform mesh size and minimal curvature of the profiles. Changing the orientation of the starting remeshed grid configuration, different solutions can be generated for the same reference surface. The corresponding plane grid geometries have been defined by means of CAD.

The advantage of this method is that by the optimisation of the initial bending stresses, the curvatures of the profiles can be modified in all directions, which helps to stay nearer to the reference surface.

*Structural Analysis by means of FEM-Modelling*

The shaping process of the initially flat grid configurations and the load-bearing capacity of the resulting gridshell structures were analysed with a three dimensional, geometric non-linear finite element model using the FEM software Sofistik AG. The grid profiles and the post-shaping bracing were modelled using beam and cable elements respectively with their corresponding material and cross sectional properties. Pure geometric surface elements, without any influence on the structure stiffness, were defined in order to apply uniformly distributed external loads. The connections between the profiles in the primary and the secondary directions were modelled using coupling elements. Taking example of the clamping connections of the already constructed gridhshells, kinematic constraints were defined so that only the in-plane grid distortion is permitted. An iterative equation solver (conjugate gradients) was applied.

The erection process starts with a flat grid whose longitudinal edges are constrained in the X-Y plane. Tubes of approximately 5 cm diameter have been used. Then upward nodal loads are distributed on the grid surface to induce the shaping of the structure. Once the desired geometry is obtained, the edges are fixed and the stiffening cables are added in order to maintain the desired gridshell structure and to obtain shear stability. The gridshell structure can now be loaded. The analysis was performed by applying uniformly distributed snow loads of incrementing magnitude.

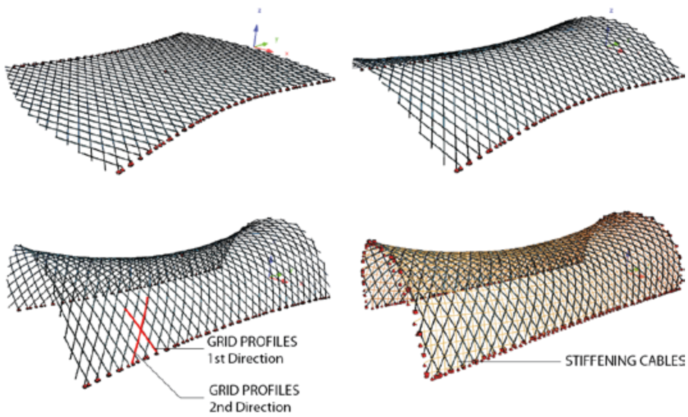


*Fig. 5 Erection process of an anticlastic gridshell modelled by FEM*

In the following figures (*Fig. 6*), the deformation (magnified by a factor of 20) under a snow load of 1.0 kN/m$^2$ are illustrated. Different deformation forms are obtained for each grid configuration.

The gridshell with predominantly acute angles between the profiles exhibits a more arch-like deformation as the gridshell contains a major number of transversal profiles which activate the arch-like structural behavior. The deformation figure consists of a single-buckle shape where the maximum deformations are concentrated in the middle of the surface. By increasing the angle between the profiles, the arch-effect decreases and a combination of arch and shell behaviors can be noticed. Indeed, the gridshell with predominantly right angles exhibits a double-buckle deformation where the central surface area sinks uniformly and the longitudinal edges, where the maximum deformations are located, buckle outwards. In the third gridshell, with predominantly obtuse angles and a major number of longitudinal profiles, the lateral edges buckle inwards stiffening the rest of the structure which sinks uniformly and lower than the first ones.

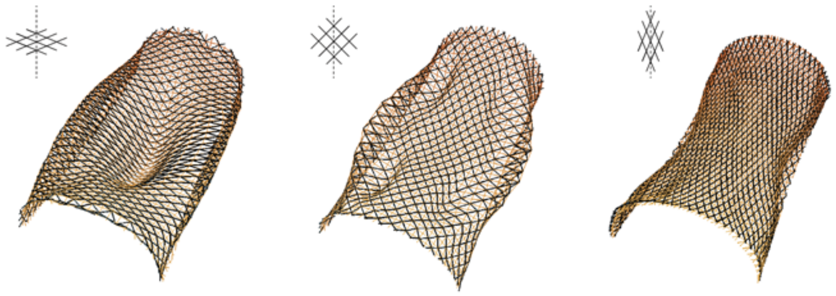

*Fig. 6 Deformation shapes under snow loads*

In the figure Fig. 7 (*left*) the maximum nodal displacements of the gridshell structures under increasing uniformly distributed surface loads are shown. One can see that by increasing the angle between the profiles the stiffness of the structure increases which results from the different deformation shapes the gridshells acquire. For a 1.0 load factor, the maximum nodal displacements of the gridshells with predominantly acute and right angles are about 6 and 3 times higher than the maximum deformations of the gridshell with obtuse angles.

Fig. 7 (*right*) shows the corresponding maximum von Mises stresses of the gridshells. Extreme values on the edges, due to geometric problems on the FEM-model, have been neglected. One can see that gridshells with lower angles, and consequently a major number of more curved transversal profiles, start with higher initial bending stresses. By the first loading factors, it can be observed that these initial bending stresses predominate and evolve relative slowly. At a certain load level, stresses due to external loads prevail and a relative linear evolution starts.

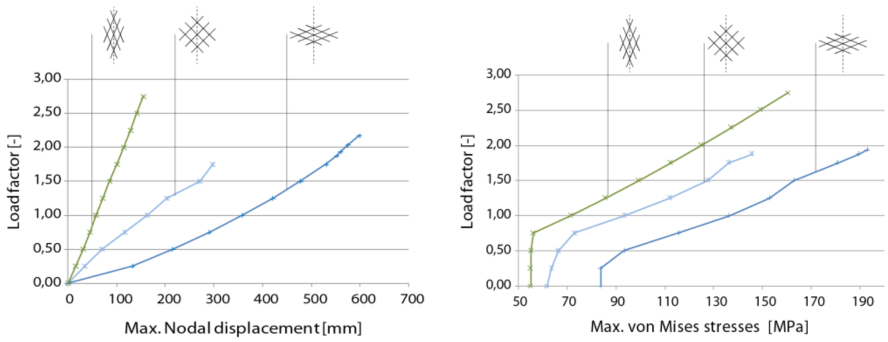


*Fig. 7 Maximum nodal displacement and von Mises stresses by increasing uniformly distributed load*

## 4. Conclusion

The final geometry of highly elastic gridshells results from a shaping process where the grid members are progressively elastically bent. In order to obtain the resulting gridshell geometry, several research studies have developed formfinding methods reproducing this erection process. Generally, these methods generate gridshells with an arbitrary orientation of the grid profiles. However, the grid orientation has an important influence not only on the post-shaping bending stresses but also on the structural behaviour of the gridshells.

The structural consequences of the grid orientation have been analysed for an anticlastic surface. Firstly, different grid configurations have been generated by means of an algorithmic calculation where the initial profiles curvatures, and consequently the initial bending stresses, tend to be minimized. Secondly, the shaping process and the load-bearing behaviour of the grids have been modelled by means of finite element methods.

The analysis show that, depending on the orientation of the grid profiles, the grid configurations adopt different deformation shapes under uniformly distributed loads, which results in variable maximum nodal displacements and von Mises stresses. Current studies analyse this structural effect for a variety of surface geometries and under diverse loading cases.

In addition, the potential of using NFRPs as environmentally-friendly alternative for gridshell structures has been studied. When regarding the bending stiffness of the profiles, NFRPs appear to be a lightweight material of choice.

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