



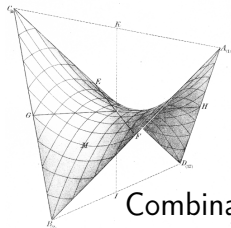
Constructing discrete minimal surfaces of s-conical type from orthogonal circle patterns: examples

Stefan Sechelmann

Institut für Mathematik, TU-Berlin

joint work with

Alexander I. Bobenko and Tim Hoffmann and Benno König



(Classical) Smooth Minimal Surface



Combinatorics of Curvature Lines / Fundamental Piece



Gauß-Image of a Fundamental Piece / Boundary Data



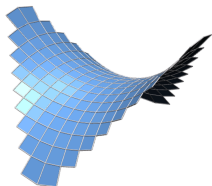
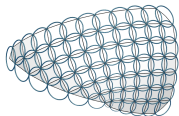
Spherical Circle Pattern



Discrete Conical Gauß Map



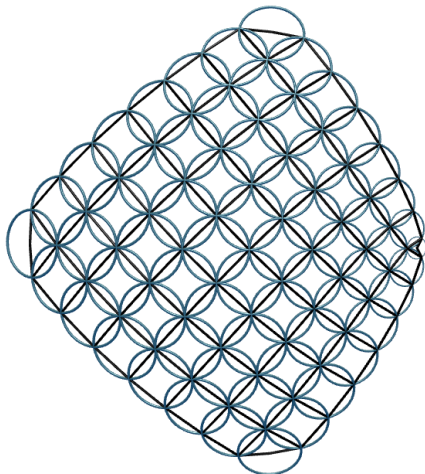
Discrete Conical Minimal Surface



schwarzp-complete-fundamental

schwarzp-fundamental-gauss

Gauß image of a fundamental piece of the Schwarz-P surface has spherical boundary angles of $\frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2}, \frac{2\pi}{3}$.

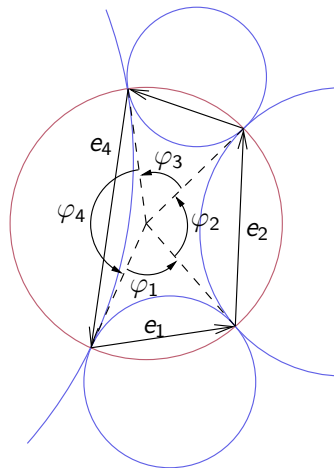
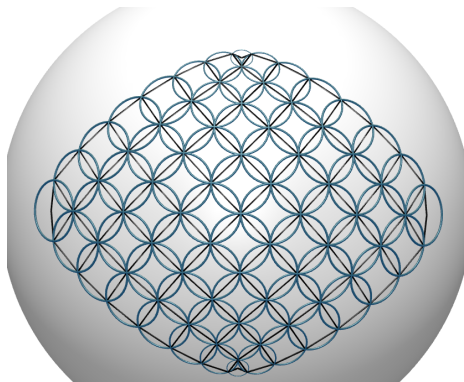


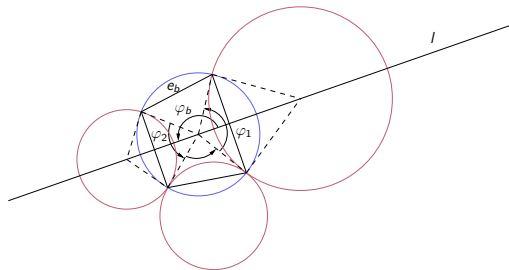
Spherical circle pattern with boundary angles $\frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2}, \frac{2\pi}{3}$ bounded by great circles.

Spherical circle patterns are the minimizer of a functional.

$$E_{sph}(\rho) = \sum_{e_{ij}} \dots + \sum_{f_i} \Phi_i \rho_i$$

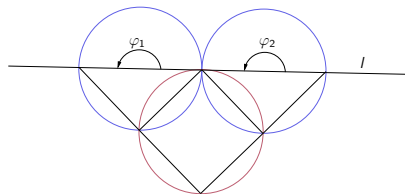
$$\frac{\partial E_{sph}}{\partial \rho_i} = \Phi_i - \sum_{e \in f_i} \varphi_e$$





Planar Curvature Line

schoenl6-circles



Straight Asymtotic Line

Gauß image bounded by great circular arcs

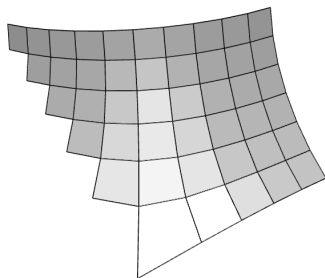
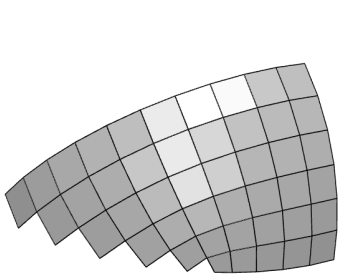
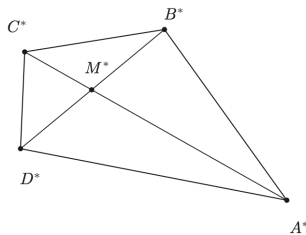
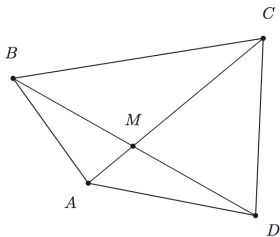
The Gauß map of a conical minimal surface is a conical mesh and has the Koenigs property.

Conical property: faces at a vertex are tangent to cone of revolution

Koenigs property: diagonal intersections are coplanar

conical-example

Calculate the Koenigs dual from the conical Gauß image.



Schwarz-P

schwarzp-complete-fundamental

schwarzp-fundamental-gauss

boundary angles: $\frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2}, \frac{2\pi}{3}$

Schwarz-P fundamental piece

schwarzp-circlepattern

schwarzp-gauss-conical

Schwarz-P dual and discrete surface

schwarzp-fundamental-conical

schwarzp-conical

Neovius' surface

neovius-complete-smooth

neovius-gauss-smooth

boundary angles: $\frac{\pi}{2}, \frac{3\pi}{4}, \frac{2\pi}{3}, \frac{3\pi}{4}$

Neovius' surface fundamental piece

neovius-circlepattern

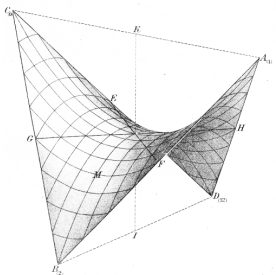
neovius-gauss-fundamental

Neovius' surface dual and discrete surface

neovius-fundamental

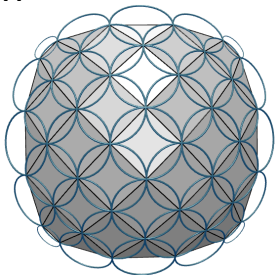
neovius-complete

Quadrilateral Boundary - straight asymptotic lines



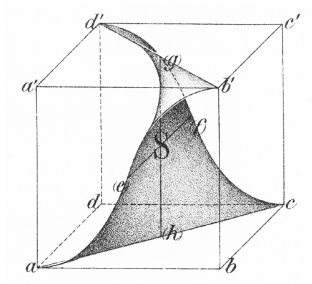
[1]

quadboundary-mesh



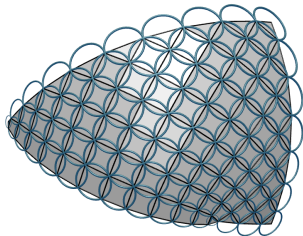
[1] H.A.Schwarz Gesammelte Mathematische Abhandlungen, Springer, Berlin 1890

Gergonnes surface - mixed boundary conditions.



gergonne

[1]



[1] H.A.Schwarz Gesammelte Matematische Abhandlungen, Springer, Berlin 1890

Catenoid type surfaces, Schwarz-H and Schoen I6

schoenI6-surface

schwarzH-surface